Preprocessing in MaxSAT Solving

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Work done with Matti Järvisalo, Tuukka Korhonen, Paul Saikko, Marcus Leivo

Maximum Satisfiability

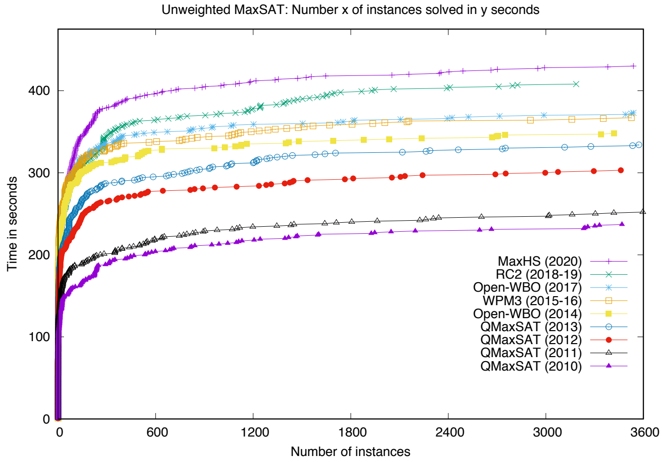
Plot by Ruben Martins

Maximum Satisfiability—MaxSAT

- Builds on the success story of SAT solving
- Great recent improvements in practical solver technology
- Expanding range of real-world applications

Solver Performance

Plot by Ruben Martins



Comparing some of the best solvers from 2010–2020:

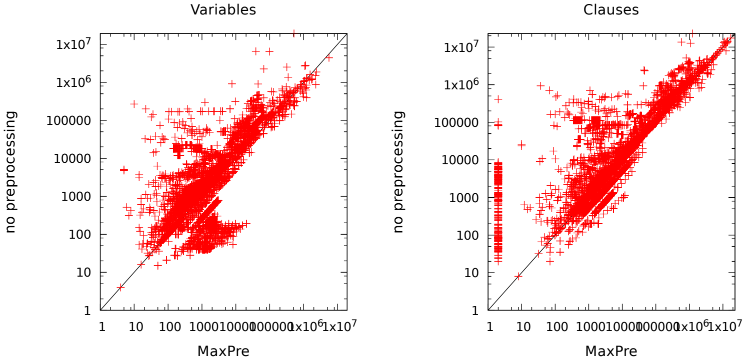
In 2020: 81% more instances solved than in 2010!

Solver Performance

An increasing number of MaxSAT solvers make use of different kinds of preprocessing.

MaxSAT preprocessing

[Korhonen, Berg, Saikko, and Järvisalo, 2017]



5425 MaxSAT instances collected and made available by the 2008–2016 MaxSAT Evaluations.

Outline

1. Basic concepts
2. MaxSAT preprocessing:
   - in practice
   - in theory
3. Conclusions

MaxSAT: Basic Definitions

MaxSAT

INPUT: a set of clauses F. (*a CNF formula*)

TASK: find τ s.t. $cost(\tau) = \sum_{C \in F} \tau(C)$ is maximized.

find a truth assignment that satisfies a maximum number of clauses

This is the standard definition, much studied in Theoretical Computer Science.

- Often inconvenient for modelling practical problems.

Central Generalization of MaxSAT

Partial MaxSAT

INPUT: sets H and S of hard and soft clauses
TASK: find model τ of H s.t:

$$cost(\tau) = \sum_{C \in S} \tau(C) \text{ is maximized}$$

find a truth assignment that satisfies all hard clauses and a maximum sum-of-weights of soft clauses

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find a truth assignment that satisfies all hard clauses and a maximum sum-of-weights of soft clauses

Note: *Can have weights on soft clauses*

Example

$$H = \{(x_1 \vee y), (\neg y \vee z),$$
$$(x_2 \vee y), (\neg y, \neg x_2)\}$$

$$S = \{(\neg x_1), (\neg x_1 \vee \neg x_2), (\neg z)\}$$

Example

$$H = \{(x_1 \vee y), (\neg y \vee z),$$
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$$\tau = \{\neg x_1, \neg x_2, y, z\}$$

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$$(x_2 \vee y), (\neg y, \neg x_2)\}$$

$$S = \{(\neg x_1), (\neg x_1 \vee \neg x_2), (\neg z)\}$$

$$cost(\tau) = 1$$

Context - MaxSAT solving

Branch & Bound MaxSAT solving

- Can be effective of small-but hard & randomly generated instances

SAT-based MaxSAT algorithms

- **Make extensive use of SAT solvers.**
- **Reduce MaxSAT solving into a sequence of satisfiability queries.**

Context - MaxSAT preprocessing

Preprocessing based on MaxSAT resolution

- Used in some B&B MaxSAT solvers
- [Heras and Bañeres, 2010; Argelich, Li, and Manyà, 2008]

SAT-based preprocessing in MaxSAT

- **Lift preprocessing rules (BVE, SE, BCE, . . .) from SAT solving to MaxSAT solving.**
- **[Belov, Morgado, and Marques-Silva, 2013; Berg and Järvisalo, 2016; Berg, Saikko, and Järvisalo, 2015; Korhonen, Berg, Saikko, and Järvisalo, 2017; Leivo, Berg, and Järvisalo, 2020; Paxian, Raiola, and Becker, 2021]**

The Nuts and Bolts of SAT-based MaxSAT preprocessing

Motivation

- SAT-based MaxSAT solvers build heavily on SAT-solvers.
- Preprocessing is important in SAT solving.

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- Preprocessing is important in SAT solving.

Central Question

Can we leverage the work on SAT preprocessing in MaxSAT?

Preprocessing a SAT instance F

F

Preprocessing a SAT instance F

$$F \xrightarrow{\mathbf{1}} pre(F)$$

1. Preprocess
   - To obtain preprocessed instance $pre(F)$

Preprocessing a SAT instance F

$$F \xrightarrow{\mathbf{1}} pre(F) \xrightarrow{\mathbf{2}} \tau^{p}(pre(F)) = 1$$

1. Preprocess
   - To obtain preprocessed instance $pre(F)$
2. Solve
   - To obtain solution τ^{p} to $pre(F)$

Preprocessing a SAT instance F

$$F \xrightarrow{\mathbf{1}} pre(F) \xrightarrow{\mathbf{2}} \tau^p(pre(F)) = 1 \xrightarrow{\mathbf{3}} \tau(F) = 1$$

1. Preprocess
   - To obtain preprocessed instance $pre(F)$
2. Solve
   - To obtain solution τ^p to $pre(F)$
3. Reconstruct
   - To obtain solution τ to F

Does not work for MaxSAT

At least not directly

$$H = \{(x_1 \vee y), (\neg y \vee z),$$
$$(x_2 \vee y), (\neg y, \neg x_2)\}$$

$$S = \{(\neg x_1), (\neg x_1 \vee \neg x_2), (\neg z)\}$$

Does not work for MaxSAT

At least not directly

$$\xrightarrow{\textbf{BVE}}$$

$$H = \{(x_1 \vee y), (\neg y \vee z), (x_2 \vee y), (\neg y, \neg x_2)\}$$

$$pre(H) = \emptyset$$

$$S = \{(\neg x_1), (\neg x_1 \vee \neg x_2), (\neg z)\}$$

$$pre(S) = \emptyset$$

Does not work for MaxSAT

At least not directly

BVE

$$\longrightarrow$$

$$H = \{(x_1 \lor y), (\neg y \lor z), (x_2 \lor y), (\neg y, \neg x_2)\}$$

$$S = \{(\neg x_1), (\neg x_1 \lor \neg x_2), (\neg z)\}$$

$$\tau^p = \{x_1, x_2, \neg y, \neg z\}$$

$$pre(H) = \emptyset$$

$$pre(S) = \emptyset$$

$$cost(\tau^p) = 0$$

Does not work for MaxSAT

At least not directly

Reconstruct

$\longleftarrow$

$\tau = \{x_1, x_2, \neg y, \neg z\}$	$\tau^p = \{x_1, x_2, \neg y, \neg z\}$
$H = \{(x_1 \vee y), (\neg y \vee z), (x_2 \vee y), (\neg y, \neg x_2)\}$	$pre(H) = \emptyset$
$S = \{(\neg x_1), (\neg x_1 \vee \neg x_2), (\neg z)\}$	$pre(S) = \emptyset$
$cost(\tau) = 2$	$cost(\tau^p) = 0$

Does not work for MaxSAT

At least not directly

Reconstruct

$\longleftarrow$

$\tau^p = \{x_1, x_2, \neg y, \neg z\}$

$H = \{(x_1 \vee y), (\neg y$

$(x_2 \vee y), (\neg y$

Problem:
preprocessing preserves satisfiability
not the cost of models

$S = \{(\neg x_1), (\neg x_1 \vee \neg x_2), (\neg z)\}$

$pre(S) = \emptyset$

$cost(\tau^p) = 0$

Solution - Labels

[Belov, Morgado, and Marques-Silva, 2013]

$F = (H, S)$

Solution - Labels

[Belov, Morgado, and Marques-Silva, 2013]

$$F = (H, S) \xrightarrow{\mathbf{1}} (H, S^E)$$

1. Label
   - $S^E = \{C \vee l_C \mid C \in S\}$

Solution - Labels

[Belov, Morgado, and Marques-Silva, 2013]

$$F = (H, S) \xrightarrow{\mathbf{1}} (H, S^E) \xrightarrow{\mathbf{2}} F^P = (pre(H \cup S^E), S^L)$$

1. Label
   - $S^E = \{C \vee l_C \mid C \in S\}$
2. Preprocess
   - $pre(H \cup S^E)$, clauses remaining after preprocessing*.
   - $S^L = \{(\neg l_C) \mid C \in S\}$

Solution - Labels

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$$\downarrow \mathbf{3}$$

$$cost(\tau^P) = cost(F^P)$$

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   - τ^P optimal to F^P
4. Reconstruct
   - τ optimal to F

Solution - Labels

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$$F = (H, S) \xrightarrow{\mathbf{1}} (H, S^E) \xrightarrow{\mathbf{2}} F^P = (pre(H \cup S^E), S^L)$$

$$\Big\downarrow \mathbf{3}$$

$$cost(\tau) = cost(F) \xleftarrow[\mathbf{4}]{} cost(\tau^P) = cost(F^P)$$

Note:
Labels also called: *relaxation variables*, *reification variables*, *blocking variables*, ...

Example

$$H = \{(x_1 \vee y), (\neg y \vee z),$$
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Example

$$\xrightarrow{\textbf{Label}}$$

$$H = \{(x_1 \vee y), (\neg y \vee z), (x_2 \vee y), (\neg y, \neg x_2)\}$$

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$$H = \{(x_1 \vee y), (\neg y \vee z), (x_2 \vee y), (\neg y, \neg x_2)\}$$

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1. Label

Example

Preprocess

$\longrightarrow$

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$$pre(H \cup S^E) = \{(l_2 \vee l_3), (l_1 \vee l_3)\}$$

$$S^E = \{(\neg l_1), (\neg l_2), (\neg l_3)\}$$

1. Label
2. Preprocess $H \cup S^E$

Example

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$\tau^P = \{\neg l_1, \neg l_2, l_3\}$

$pre(H \cup S^E) =$
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$cost(\tau^P) = 1$

1. Label
2. Preprocess $H \cup S^E$
3. Solve

Example

Reconstruct

$\longleftarrow$

$\tau = \{\neg x_1, \neg x_2, y, z\}$

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$(x_2 \vee y), (\neg y, \neg x_2)\}$

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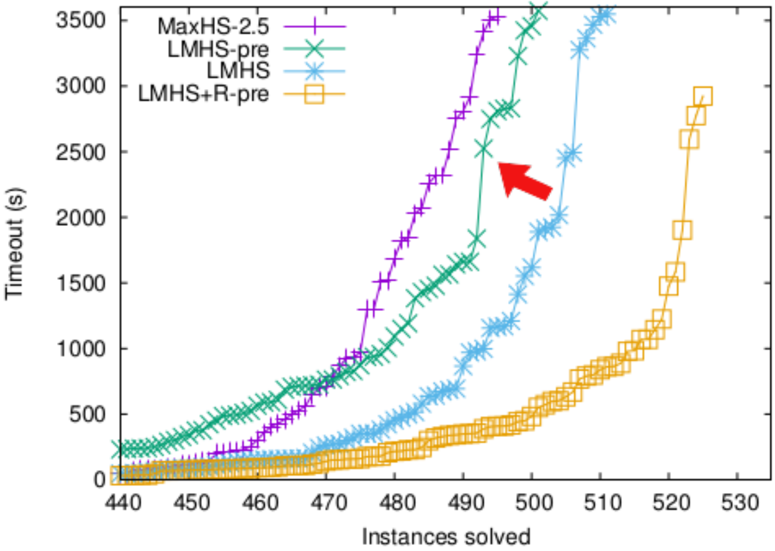
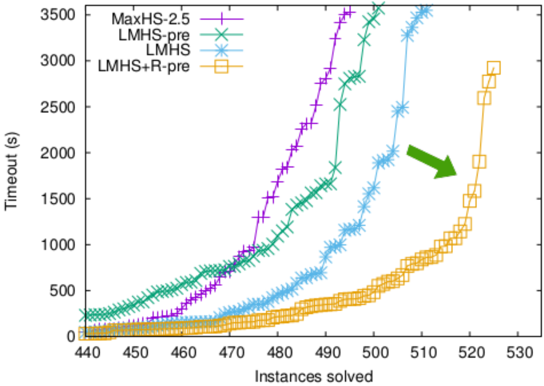
However

[Berg, Saikko, and Järvisalo, 2015]



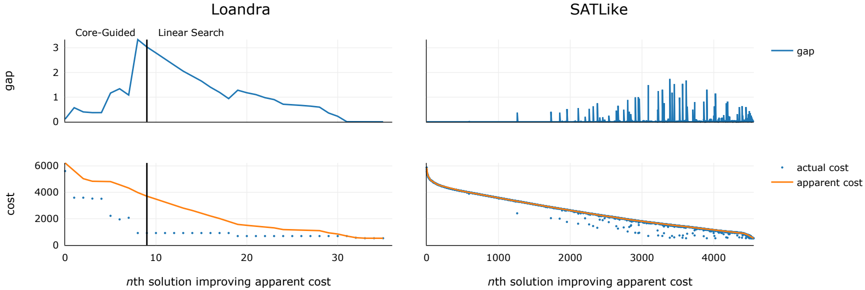
Solution



- Modify the solver to use labels directly as assumptions,
- Today implemented by many available solvers
  - *but not necessarily all*.

We are not quite done....

[Leivo, Berg, and Järvisalo, 2020]



- Intermediate solutions central in many solvers.
- Costs of intermediate solutions of $pre(F)$ are often misrepresented.

Beyond SAT-based preprocessing

Many solvers incorporate various "preprocessing-like" heuristics during search:

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- Stratification & Hardening [Ansótegui, Bonet, Gabàs, and Levy, 2012]
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Beyond SAT-based preprocessing

Many solvers incorporate various "preprocessing-like" heuristics during search:

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  - use bounds on $cost(F)$ in order to harden soft clauses.
- Reduced cost fixing [Bacchus, Hyttinen, Järvisalo, and Saikko, 2017]
  - use reduced cost of soft clauses to fix them to true or false.

Beyond SAT-based preprocessing

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  - use bounds on *cost*(F) in order to harden soft clauses.
- Reduced cost fixing [Bacchus, Hyttinen, Järvisalo, and Saikko, 2017]
  - use reduced cost of soft clauses to fix them to true or false.
- and others: [Paxian, Raiola, and Becker, 2021; Ignatiev, Morgado, and Marques-Silva, 2019; Martins, Manquinho, and Lynce, 2013]

Take Home Message

Preprocessing can be effective in MaxSAT

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Preprocessing can be effective in MaxSAT

as long as your solver supports it

The Theory of SAT-based MaxSAT preprocessing

Central Concepts

Cores & MUSes

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- $\textsc{mus}(F)$ = set of all MUSes of F.

Central Concepts

Cores & MUSes

A set $\kappa \subset S$ of an instance (H, S) is:

- A core if $H \wedge \kappa$ is UNSAT
- An MUS if $H \wedge \kappa_s$ SAT for all $\kappa_s \subset \kappa$.
- $\textsc{mus}(F) =$ set of all MUSes of F.

Why relevant for MaxSAT?

Every solution to (H, S) falsifies at least one clause from each MUS.

What we know

Theorem

[Belov, Morgado, and Marques-Silva, 2013]

Preprocess F with BVE, BCE, SE or SSR to obtain $pre(F)$.
Then:

$$\textsc{mus}(F) = \textsc{mus}(pre(F))$$

What we know

Theorem

[Belov, Morgado, and Marques-Silva, 2013]

Preprocess F with BVE, BCE, SE or SSR to obtain $pre(F)$.
Then:

$$\textsc{mus}(F) = \textsc{mus}(pre(F))$$

Consequence

[Berg and Järvisalo, 2016]

The best case number of iterations (SAT-solver calls) of many MaxSAT solvers when solving F and $pre(F)$ are equal.

More Generally

[Berg and Järvisalo, 2019; Järvisalo, Heule, and Biere, 2012]

Max-RAT

- Extension of resolvent asymetric tautologies (RAT) to MaxSAT.
- (Informally) C is Max-RAT if it is RAT on a non-label variable.

More Generally

[Berg and Järvisalo, 2019; Järvisalo, Heule, and Biere, 2012]

Max-RAT

- Extension of resolvent asymetric tautologies (RAT) to MaxSAT.
- (Informally) C is Max-RAT if it is RAT on a non-label variable.

Theorem

Preprocess F with any techniques corresponding to the addition and removal of Max-RAT clauses to obtain $pre(F)$.
Then:

$$\textsc{mus}(F) = \textsc{mus}(pre(F))$$

More Generally

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Max-RAT

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Theorem

Preprocess F with any techniques corresponding to the addition and removal of Max-RAT clauses to obtain $pre(F)$.
Then:

$$\textsc{mus}(F) = \textsc{mus}(pre(F))$$

so the best-case number of iterations of solvers are equal

Take-Home Message

Significantly altering the execution of MaxSAT solvers via preprocessing requires affecting the MUSes

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Significantly altering the execution of MaxSAT solvers via preprocessing requires affecting the MUSes

Liftings of commonly used techniques in SAT do not do this

Beyond techniques from SAT

- **hardening, reduced cost fixing, subsumed label elimination**

  [Bacchus, Hyttinen, Järvisalo, and Saikko, 2017; Ansótegui, Bonet, Gabàs, and Levy, 2012; Berg, Saikko, and Järvisalo, 2016]

  - Can fix soft clauses in MUSes.
- **intrinsic at-most-one constraints**

  [Ignatiev, Morgado, and Marques-Silva, 2019]

  - Can alter MUSes.

Take-Home Message

Many effective preprocessing rules for MaxSAT have been proposed as part of solver heuristics
Unifying the theory underlying the existing methods could lead to new insights & more effective solvers

Preprocessing for MaxSAT- Summary

- Preprocessing for MaxSAT can be effective
  - Requires careful integration with the solver.
- The field is divided.
  - Unifying the theory underlying existing techniques central for further improvements.

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Available software

- MaxPRE: `https://github.com/Laakeri/maxpre`

  [Korhonen, Berg, Saikko, and Järvisalo, 2017]
- Coprocessor [Manthey, 2012]

Further Reading and Links

Talks at the Simons Institute

- Fahiem Bacchus on incremental SAT and MaxSAT on April 1st.
- Jeremias and Matti Järvisalo on MaxSAT on April 13th.

Surveys

- "Maximum Satisfiability" by Bacchus, Järvisalo & Martins
  - Chapter in forthcoming vol. 2 of Handbook of Satisfiability
  - Preprint available.
- Somewhat older surveys:
  - Handbook chapter on MaxSAT: [Li and Manyà, 2009]
  - Surveys on MaxSAT algorithms:

    [Ansótegui, Bonet, and Levy, 2013]

    [Morgado, Heras, Liffiton, Planes, and Marques-Silva, 2013]

MaxSAT Evaluations

https://maxsat-evaluations.github.io

Most recent report: [Bacchus, Järvisalo, and Martins, 2019]

Thank you for attending!

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