Solving Optimization Problems via Maximum Satisfiability: Encodings and Re-Encodings

Jeremias Berg

ACP Doctoral Research Award @ CP2020

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HELSINKI INSTITUTE FOR INFORMATION TECHNOLOGY

Who am I?

www.jeremiasberg.com

- MSc. in mathematics.
- Graduated with distinction from the University of Helsinki in June 2018.
  - Supervisor: **Prof. Matti Järvisalo**.
  - PhD Committee:
    - **Prof. Peter Stuckey**,
    - **Prof. Lakhdar Sais** and
    - **Prof. Inês Lynce** as the opponent.

Thanks to Matti Järvisalo and Ruben Martins for providing material for these slides

My Doctoral Research

Developing Maximum Satisfiability (MaxSAT) for solving real-world optimization problems.

More specifically:

- Understanding and developing solver independent preprocessing.
- Applying MaxSAT toward NP-hard data analysis problems.

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Outline

The talk is divided into two parts:

1. Maximum Satisfiability in real world optimization.
2. Main results of doctoral research.

Maximum Satisfiability

MaxSAT

Exact Boolean optimization paradigm:

- Based on propositional logic
  - i.e. weighted Boolean combinations of binary variables.
- Builds on the success of SAT.
- Expanding range of real-world applications.
- Effective and often orthogonal alternative to e.g.
  - mixed integer programming
  - finite-domain constraint optimization
  - and others

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Applications of MaxSAT

Drastically increasing number of successful applications

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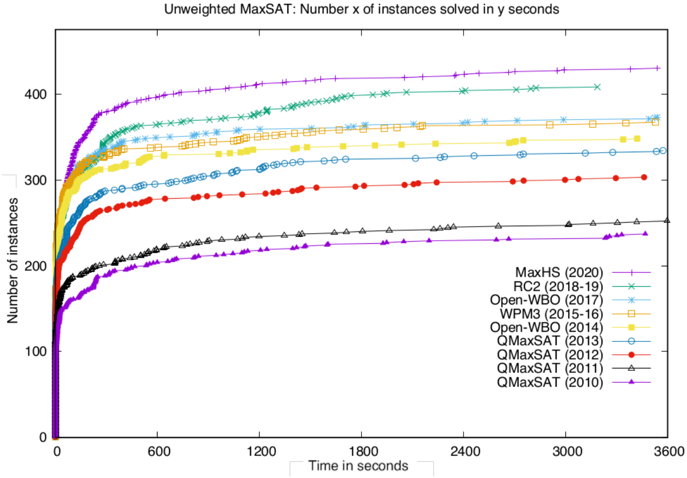
Central to the increasing success:
Advances in solver technology

Evolution of MaxSAT solvers

Plot provided by Ruben Martins



83% more instances solved in 10 years

MaxSAT solving

Focus of the thesis (and these applications):
SAT-based MaxSAT algorithms

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In more detail:

- Algorithms that make iterative use of SAT solvers.
- Three central ones:
  - *model-improving*,
  - *core-guided* and
  - *implicit hitting set* based.
- Central results of thesis applicable to all.

Results of Thesis Research

Thesis

Six core publications aiming to improve MaxSAT-based solution methods to combinatorial optimization problems.

- Part 1: MaxSAT solving techniques, specifically preprocessing
  IJCAI 2015, ICTAI 2015, CP 2016, ECAI 2016
- Part 2: MaxSAT based approaches to solving data analysis problems
  AISTATS2014, AIJ 2015

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Part 1: MaxSAT Preprocessing

Main results

1. Integrating preprocessing and solving requires care.
   - Straightforward way degrades performance.
   - Our more careful way improves performance.
2. Liftings of SAT preprocessing rules have limited effect on MaxSAT algorithms.
   - Inability of affecting small unsatisfiable subsets of constraints is a fundamental limitation.

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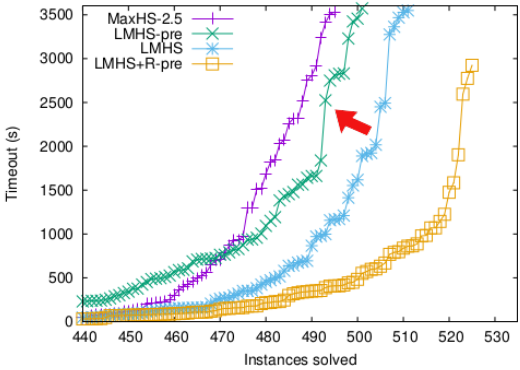

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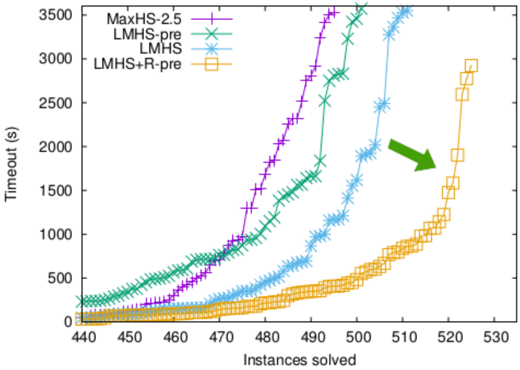

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Subsequently

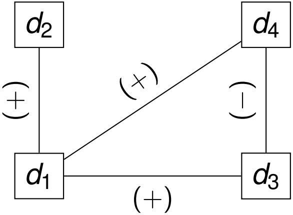
- Increasing number of solvers in MSEs use preprocessing.
- Improved theoretical understanding of MaxSAT preprocessing.

[Berg and Järvisalo(2019)]

Part 2: MaxSAT for Data Analysis

Correlation Clustering

- A set of datapoints to be clustered.
  - Pairwise similarity measure over the points.
- Want a clustering that correlates well with the measure.
  - Similar points $\rightarrow$ same cluster.
  - Dissimilar points $\rightarrow$ different clusters.
  - **Note:** No restriction on number of clusters.



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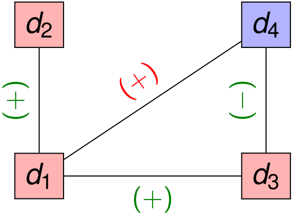
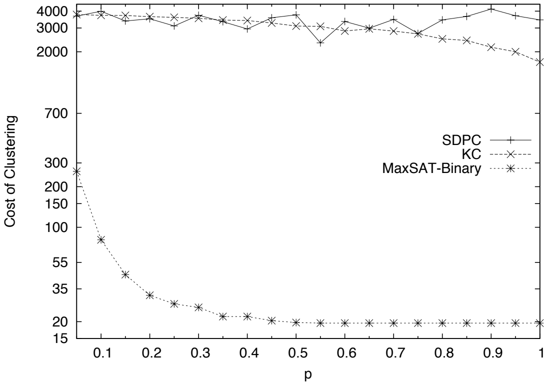


Applications in: biosciences, social network analysis, information retrieval, bioinformatics and microarray data analysis . . . .

Correlation Clustering

Main results

- Solutions obtained with MaxSAT are significantly better than the ones obtained by specialized approximative algorithms.

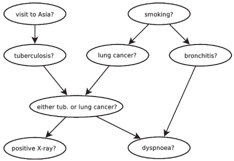


Bounded Treewidth Bayesian Network Structure Learning

BTBNSL

A	S	T	L	B	T&L	X	D
0	1	0	1	0	1	1	1
0	0	1	1	0	1	0	1
1	1	0	1	1	0	0	1
1	0	0	1	1	1	1	1
1	1	1	1	1	1	0	1
1	1	1	1	0	0	1	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

$\rightarrow$



$\rightarrow$

$$P(S \mid T, L, B) = ?$$
$$P(X \mid D, A, B) = ?$$
$$P(A \mid T, L, B) = ?$$
$$\vdots$$

- Starting from observations (data) learn a Bayesian Network that explains the data as well as possible.
  - NP-hard
- Restrict the treewidth of the learned structure.
  - NP-hard

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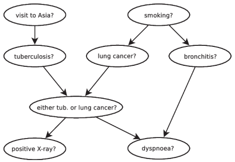
$\longrightarrow$



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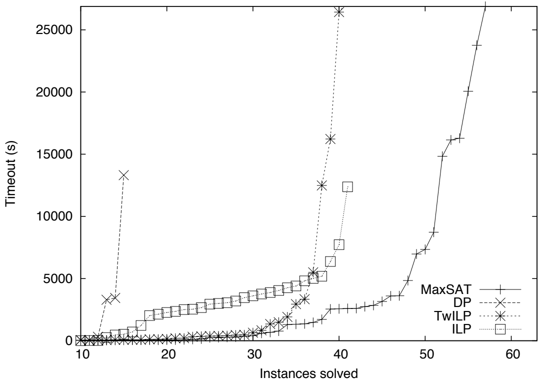
BTBNSL

Main Results

- MaxSAT scales better than other declarative approaches .
- Adding the (highly non-trivial) treewidth constraint to the encoding improves overall performance.



Additional Results of Thesis Research

- Seven related papers developing MaxSAT algorithms and applications,
- Contributions to open-source software

  [Korhonen et al.(2017)Korhonen, Berg, Saikko, and Järvisalo, Saikko et al.(2016)Saikko, Berg, and Järvisalo]
- Openly available benchmark sets [Berg et al.(2018)Berg, Hyttinen, and Järvisalo]
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Recent Highlights

- Hybrid algorithms for MaxSAT:
  - Loandra - One of best incomplete solvers in recent MSEs.
    [Berg et al.(2019)Berg, Demirovic, and Stuckey]
  - Best paper award at SAT 2020. [Berg et al.(2020)Berg, Bacchus, and Poole]

Conclusions

Take Home Message:
Modern MaxSAT has developed into an effective and competitive approach to exact optimization.

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Future research directions:

- Better hybrid search techniques (e.g. inprocessing).
- Incremental applications of solvers to further improve scalability.
- Improved understanding of the relationship between applications and optimization paradigms.

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Thank you!

I would like to thank:

- the Association for Constraint Programming,
- my supervisor Matti Järvisalo,
- the people involved in the nomination:
  - Inês Lynce,
  - Peter Stuckey,
- as well as all my co-authors and collaborators:
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  - Brandon Malone
- and all of you for watching.

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